Table 1 Screen performance parameters

Screen	Bubble Point Ft, LH_2	α	Flowthrough Parameters β	Roughness ϵ, in.	Weight lb/100 ft^2 (stainless steel)
$325\times 2{,}300$	1.580	3.2	0.19	0.0005	10.9
$200\times 1{,}400$	1.108	4.2	0.20	0.0008	18.6
720×140	0.580	11.0	0.47	0.00215	14.2
165×800	0.403	3.3	0.17	0.001	16.3
50×250	0.224	13.5	0.26	0.00225	23.2
24×110	0.1105	8.61	0.52	0.00525	63.5
500×500	0.540	5.7	0.65 (0.77)[a]	0.001	3.4
150×150	0.151	5.7	0.50(0.50)[a]	0.0026	7.0
60×60	0.0754	5.7	0.40 (0.61)[a]	0.0075	23.9
40×40	0.0559	5.7	0.60 (0.52)[a]	0.01	27.9

[a] Based on Euler number $=(S/1-S)^2$.

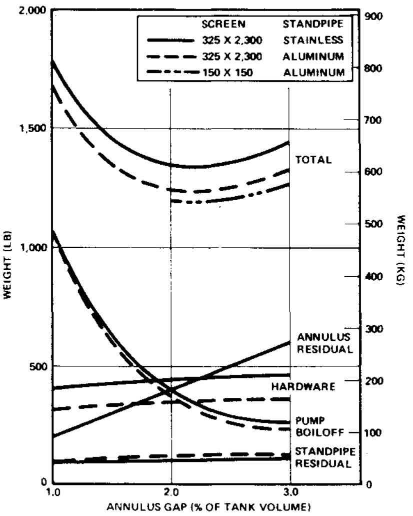

Fig. 2 Weight optimization for 5000 ft^3 LH_2 tank for 300 day mission.

are contained in Ref. 8. The important characteristics of the 10 screens tested are shown in Table 1.

Parametric Analysis

Because the screen annulus remains full of propellant at all times, it represents an undesirable residual and weight penalty which should be minimized. However, as the annulus gap is made smaller, the pressure loss for flow through the annulus increases, with two undesirable results. First, the pressure loss during outflow may exceed the screen bubble point resulting in screen breakdown and bubble ingestion into the outflowing liquid. Second, during coast, the pressure loss for the TVS flow is increased, resulting in higher pump power requirements, and thus higher heat input and equivalent boiloff of the cryogen. These effects were studied parametrically for a range of flow conditions and g-levels for LH_2 tanks from 50-5000 ft^3 and for liquid oxygen (LO_2) tanks from 5-500 ft^3, for 30- and 300-day orbital storage missions and for all 10 screens.

The screens and annulus gaps were selected so that half the screen bubble point would not be exceeded during outflow (a safety factor of 2). Then the annulus gap and central standpipe size were optimized for minimum residual and coast heating boiloff. The effects of annulus residual and pump boiloff counteracted each other as a function of annulus gap to give a minimum system weight value for annulus gap, as shown typically in Figure 2 for 5000 ft^3 LH_2 tank with a 300-day coast mission.

Conclusions

The parametric analysis, based on unique experimental data on screen properties, has shown that the integrated pump TVS/wall screen liner system is fluid-dynamically feasible for LH_2 and LO_2 tanks over a wide range of sizes and operating conditions. Further work will be necessary to determine thermodynamic feasibility, define fabrication concepts, and compare the pump TVS/wall screen liner system to other methods of orbital storage and transfer, such as supercritical or propulsion-accelerated systems.

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Optimum Orbital Control Using Solar Radiation Pressure

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Nomenclature

a,e	= orbit's semimajor axis and eccentricity
m	= mass of satellite
n	= mean angular speed in orbit
p	= solar pressure on a perfectly reflecting surface placed normal to the sun rays
C_q,S_q,T_q	= cos (q), sin(q), tan(q), respectively

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I. Introduction

COMMUNICATION satellites demand precise orbital control. Launch errors and perturbations caused by the internal, as well as environmental causes, must be corrected. Active systems, i.e., micro-thrusters, are normally used, which, though efficient, suffer from the disadvantages of limited life, low reliability, and high cost. Therefore, some alternative controllers—passive or semipassive—should be considered.

The solar radiation pressure is a dominant source of orbital perturbation for larger satellites.[1] A modification in design can lead to its utilization to advantage. Its application to interplanetary travel[2] and satellite attitude control[2-5] are well established. The concept is also extended to orbital corrections of near-earth satellites[6,7] as well as geostationary systems[8,9] The numerical results for a few simple schemes proposed in these studies appear to be quite interesting. It is felt that an optimum control of the actuators could improve the performance significantly. In this note, the control schemes of a panel (which can turn in and out of the orbital plane) leading to the maximum changes in various orbital elements are obtained through parametric optimization. The formulation employs Lagrange's orbital perturbation equations.[10] A few case studies show that a simple near optimal control relation leads to adequate, gentle corrections.

II. Analysis

Equations of Motion

The complete system dynamics is rather complex. As the main objective here is to study the feasibility of the concept, the forces acting on the panel alone are considered. The librational motions of the satellite are ignored, and both sides of the panel are assumed to be usable and perfectly reflective.

Using the standard geocentric ecliptic coordinate system, two successive rotations, δ in the orbital plane (pitch) and γ in the transverse direction (roll), are used to impart any desired orientation to the panel (Fig. 1). The radiation force acting on the surface of area A_p is given by

$$F=F_N=p.A_pC_\alpha\,|C_\alpha| \tag{1}$$

where the angle of incidence α can be found by successive coordinate transformations as[4]

$$C_\alpha=S_{(\omega+\theta+\delta)}S_{(\Omega-\psi)}C_iC_\gamma - C_{(\omega+\theta+\delta)}C_iC_{(\Omega-\psi)}+S_{(\Omega-\psi)}S_iS_\gamma \tag{2}$$

The forces being small, the equations of motion can be written in terms of Lagrange's relations for variations of orbital elements.[10] Such an approach was found advantageous in the case of ion-thrusting.[11] Using θ, as the independent variable, the equations are

$$a'=2F_laC_\gamma\{S_\delta+eS_{(\theta+\delta)}\} \tag{3a}$$

$$e'=F_lC_\gamma\{S_\theta C_\delta+S_\delta(eC_\theta^2+e+2eC_\theta)/(1+eC_{t\theta})\} \tag{3b}$$

$$i'=-F_l(1-e^2)S_\gamma C_{(\omega+\theta)} \tag{3c}$$

$$\Omega'=-F_l(1-e^2)S_\gamma S_{(\omega+\theta)}/S_i \tag{3d}$$

$$\omega'=2\Omega' S_i^2-F_lC_\gamma\{C_\theta C_\delta - S_\theta S_\delta(2+eC_\theta)/(1+eC_\theta)\}/e \tag{3e}$$

where primes indicate differentials with respect to θ and

$$F_l=F(1-e^2)/\{n^2am(1+eC_\theta)^2\} \tag{4}$$

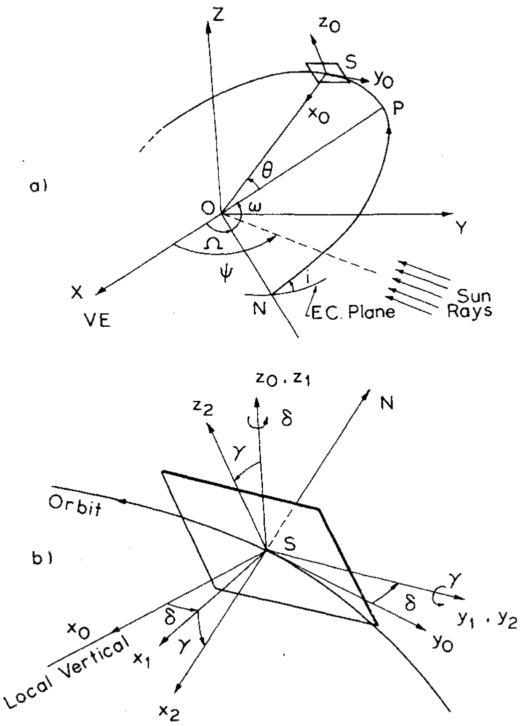

Fig. 1 Geometry of motion.

By controlling the panel angles δ and γ, the orbital elements can be changed as desired.

Optimal Control of Orbital Elements

The objective is to accomplish the corrections in the orbital elements in a minimum time. The control angles which maximize the above variations would generally lead to such an optimal performance of the system. These angles can be determined by a simple parametric optimization, i.e., by setting

$$\partial f_j/\partial\delta=0 \text{ and } \partial f_j/\partial\gamma=0 \tag{5}$$

where f_j represents the right-hand side of the relation corresponding to the jth orbital element, the change in which is to be maximized. The optimal relations for δ and γ, thus found, for a few cases are summarized here.

1) For maximum increase in the semimajor axis

$$\delta=-\phi_l-\tan^{-1}\{(A-B+CT_\gamma)D/2(AB+D^2)\} \tag{6a}$$

$$\gamma=\tan^{-1}\{[-3(A-B)\pm\sqrt{9(A-B)^2+8C^2}]/2C\} \tag{6b}$$

where

$$\phi_l=\tan^{-1}\{eS_\theta/(1+eC_\theta)\}$$

$$A=S_{(\omega+\theta+\delta)}T_{(\Omega-\psi)}C_i\;;\quad B=C_{(\omega+\theta+\delta)}\;;$$

$$C=T_{(\Omega-\psi)}S_i\;;\quad D=S_{(\omega+\theta+\delta)}$$

The previous transcendental equations would have to be solved by iteration. Also, of the possible values corresponding to the $\pm$ signs, the one ensuring positive a' should be chosen.

2) The best controls for maximizing the changes in i and Ω are

$$\delta=\tan^{-1}(-T_{(\Omega-\psi)}\cdot C_i)-(\omega+\theta) \tag{7a}$$

$$\gamma=\tan^{-1}\{(-3E\pm\sqrt{9E^2+8})/4\} \tag{7b}$$

where

$$E=-T_{(\Omega-\psi)}\cdot S_i\sqrt{1+C_i^2T^2_{(\Omega-\psi)}}$$

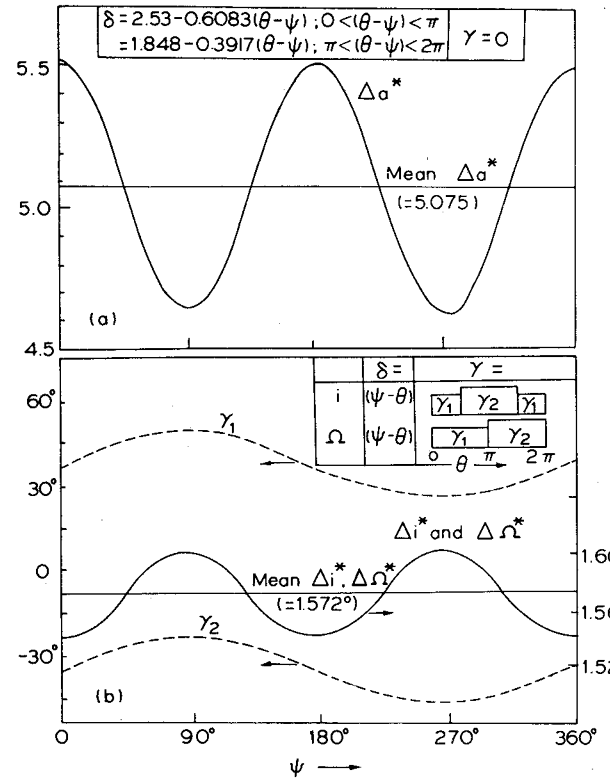

Fig. 2 Optimal correction of the orbital elements.

Again the ± sign is chosen so as to keep i' or Ω' positive. Here δ changes continuously with θ while γ takes only two fixed values.

III. Case Studies

To establish the effectiveness of the previous system, its application in the cases of an ecliptic circular and a geostationary orbit is studied. The resulting corrections are nondimensionalized with respect to the satellite and orbital parameters as

$$\Delta a^* = \Delta a / X \tag{8a}$$

$$\Delta i^* = \Delta i / (a/X) \tag{8b}$$

$$\Delta\Omega^* = \Delta\Omega / (aS_i/X) \tag{8c}$$

where Δa, Δi, $\Delta\Omega$ denote the changes in a, i, Ω per revolution and

$$X = p\, A_p / mn^2$$

Ecliptic Circular Orbit

Since $\Omega = i = \omega = e = 0$, the optimal control for correcting a is

$$\gamma_{\rm opt} = 0; \quad \delta_{\rm opt} = \tan^{-1}\{[-3T_{(\theta-\psi)} \pm \sqrt{9T^2_{(\theta-\psi)} + 8}]/4\}$$

$$(+\text{for } \pi/2 < (\theta-\psi) < 3\pi/2; \quad -\text{for } -\pi/2 < (\theta-\psi) < \pi/2) \tag{9}$$

The corresponding $\Delta a^*_{\max}$ (=5.55) is considerably larger than those achievable by any of the schemes proposed by the earlier investigators.[6-9]

The preceding control is complicated and may be difficult to implement. A bilinear control approximates it closely:

$$\delta = 2.53 - 0.6083(\theta-\psi) \quad \text{for } 0 < (\theta-\psi) < \pi$$

$$= 1.848 - 0.3917(\theta-\psi) \qquad \pi < (\theta-\psi) < 2\pi \tag{10a}$$

$$\gamma = 0 \tag{10b}$$

with this, Δa^* turns out to be 5.51 which is only 0.7% less than the $\Delta a^*_{\max}$. *Moreover, the changes in other elements are negligible.*

Geostationary Orbit

1) Corrections in a: The optimal control would call for the solution of the transcendental equation. Since the effect of low inclination is not likely to affect the performance significantly, the bilinear control given by Eq. (10) can be used. For this Δa^*, which is a function of ψ, is given in Fig. 2a. The other orbital elements remain practically unchanged.

2) Correction in i and Ω: Solution of Eq. (7) gives the required controls. The orbital inclination being low, a simplified control of δ which closely approximates the optimal is given by $\delta = \psi - \theta$. Over one revolution ψ can be taken to be constant which makes δ linear with θ. The other angle γ depends on the element to be corrected as

$$\gamma = \gamma_1 \text{ for } -\pi/2 < \theta < \pi/2 \qquad 0 < \theta < \pi$$

or

$$\gamma = \gamma_2 \; \pi/2 < \theta < 3\pi/2 \text{ for changing } i \;\; \pi < \theta < 2\pi \text{ for } \Omega \tag{11}$$

The values of γ_1, γ_2 and the possible corrections Δi^* and $\Delta\Omega^*$ are functions of the sun position as shown in Fig. 2b.

The actual corrections possible are proportional to the area to mass ratio of the satellite-panel combination. For A_p/mass = 1(m^2/kg), the previous strategies yield $\Delta a = 9.6$ km/day, $\Delta i = 1.47°$/year or $\Delta\Omega = 3.7°$/year with independent control. A comparison with the actual perburbations[8] indicates that a panel with A_p/mass ≃ 0.8 would overcome all external disturbances completely and increase the lifetime, theoretically, to infinity. This, of course, means a need for a very large panel(s). For East-West stationkeeping, however, a much smaller surface (A_p/mass ≈ 0.01) would be adequate. The "gentle" continuous control could also reduce the perturbations in orbital inclination and thereby increase the lifetime at "no cost." It is felt that the concept warrants a serious consideration.

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